DETECTING THE DYNAMICAL STATE OF THE ATMOSPHERE FROM THE ORBITAL DECAY OF THE ODERACS SPHERES

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ABSTRACT

The orbital decay curve of a satellite having constant cross-sectional area and in circular orbit can furnish valuable information regarding the dynamical state of the atmosphere. It is shown that a rectilinear decay curve having constant downward slope (zero curvature) should indicate that the atmosphere was undergoing compression during that period. A decay curve having concavity upwards (positive curvature) will strongly indicate that the atmosphere was in a contracting phase. A decay curve with downward concavity (negative curvature) may indicate an expanding, a stationary or a contracting atmosphere. This theory, when applied to the orbital decay of the Orbital Debris Radar Calibration Spheres (ODERACS) satellites, indicates that during the period from Day 90 through Day 240 in the year 1994, the atmosphere was very definitely in a compression mode. During this period, ODERACS Sphere 1 faced nearly constant densities while Sphere 6 actually encountered progressively smaller air densities as they descended. The atmospheric scale height as calculated from the orbital data of Spheres 1 and 6 diminished steadily during the same period. It is shown that Spheres 1 and 6 descended faster and slower, respectively, than the level of constant air density equal to 5×10^{-12} kg/m^3. During a brief period from Day 240 through Day 290, the atmosphere reversed to a strongly expanding mode. Thereafter, the atmosphere reverted back to a compression mode from Day 290 through Day 390, 1994.

INTRODUCTION

The orbital decay of artificial Earth satellites have provided valuable information regarding atmospheric densities, temperature and other parameters upon which most atmospheric models are now based. Further, specific information can be extracted from satellite orbits which are nearly circular. For example, Tobiska, et al. (1987) developed a 10.7 cm solar flux model for sunspot cycle 22 from the observed decay of the Solar Mesosphere Exposure Satellite, whereas Badhwar (1990) determined the exospheric temperature during the same cycle from the orbital decay of the Long Duration Exposure Facility Satellite.

This study analyzes the orbital decay of the Orbital Debris Radar Calibration Spheres (ODERACS) satellites, which were deployed on 9 February 1994 from the Space Shuttle Discovery into nearly circular orbits of 352 x 347 km (The ODERACS News Roundup, February 1994). The ODERACS consisted of six perfectly spherical metallic satellites: two 4"-diameter Aluminum spheres; two 2"-diameter Stainless Steel spheres; and two 6"-diameter Aluminum spheres. One sphere of each category had a smooth surface and the other was sandblasted. The smooth Aluminum spheres were also chrome-plated. On account of the perfectly spherical shapes, the cross-sectional areas of the ODERACS were constants and accurately known.

The present study analyzes the orbital decay of satellites having constant cross-sectional areas and in nearly circular orbits. Special attention is focussed on the significance of the slope and curvature of the orbital decay curve and their implications on the dynamic state of the atmosphere. Particular emphasis is given to finding the signatures of atmospheric compression. The theory is applied to the orbital decay of the ODERACS satellites. It is shown that from Day 90 through Day 240 during the year 1994, the atmosphere was in a compression phase.

THEORY

The theory of orbital decay is well documented in the literature. The rate of change of the semi-major axis of a satellite of mass m is given by (cf. King-Hele, 1964)

$$\dot{a} = - \frac{C_D \, a^2 \, A \, \rho \, v^3}{\mu \, m} , \tag{1}$$

where C_D is the drag coefficient, A the cross-sectional area of the satellite, v its velocity, ρ the density of the ambient air and μ the gravitational parameter of the Earth. The dot represents total differentation with respect to time. Equation (1) can be employed to calculate the density of the ambient air if the cross-sectional area of the satellite is accurately known (cf. Tobiska, et al., 1987).

For a circular orbit, we have

$$v^2 = \frac{\mu}{a} , \tag{2}$$

so that Eq. (1) takes a simpler form (cf. Badhwar, 1990)

$$\dot{a} = - \frac{C_D \, A \, a \, \rho \, v}{m} . \tag{3}$$

Since atmospheric drag reduces the eccentricity of elliptic orbits, an initially circular orbit remains circular (cf. King-Hele, 1964). For circular orbits, $a = r_o + z$, where r_o is the reference radius of the Earth and z can be taken as the mean altitude of the satellite. $\dot{a}$ is geometrically equal to the slope of the tangent to the decay curve on a plot of the semi-major axis a (or z for that matter) versus time.

As a satellite in a circular orbit descends in the atmosphere, it normally faces greater and greater atmospheric densities. The slope of the decay curve becomes progressively steeper and the curve becomes concave downwards, as a general rule. However, if a decay curve is straight over a considerable period of time, then $\dot{a}$ is zero over that period. In that case, the variation of density encountered by the satellite as a function of z is obtained from Eqs. (3) and (2):

$$\rho \propto \left(1 + \frac{z}{r_o} \right)^{-\frac{1}{2}} . \tag{4}$$

If the satellite descends from an altitude z_1 (corresponding density ρ_1) to an altitude z_2 (corresponding density ρ_2), then

$$\rho_1 = c \left(1 + \frac{z_1}{r_o} \right)^{-\frac{1}{2}} , \tag{5}$$

and

$$\rho_2 = c \left(1 + \frac{z_2}{r_o} \right)^{-\frac{1}{2}} , \tag{6}$$

c being the constant of proportionality. Dividing (6) by (5), expanding and retaining first order terms only, we have

$$\frac{\rho_2}{\rho_1} \cong 1 + \frac{z_1 - z_2}{2 \, r_o} . \tag{7}$$

Since $z_1 - z_2$ is positive, the density increases as the satellite descends. However, as $(z_1 - z_2)/r_o$ is small, the increase is a slowly varying function of the altitude drop $z_1 - z_2$.

This increase may now be compared with the natural increase in density encountered by an object descending through a stationary atmosphere. In the altitude range of low-Earth-orbit satellites, the atmospheric density decreases exponentially with height, with a scale height of H between 30 and 50 km (cf. King-Hele, 1964):

$$\rho = \rho_0 e^{-\frac{z}{H}} . \tag{8}$$

Here ρ_o is the density at a reference base level. If the object descends between altitude z_1 (corresponding density ρ_1) and altitude z_2 (corresponding density ρ_2), then

$$\rho_1 = \rho_0 e^{-\frac{z_1}{H}} , \tag{9}$$

and

$$\rho_2 = \rho_0 e^{-\frac{z_2}{H}} . \tag{10}$$

Dividing (10) by (9), expanding and retaining first order terms only,

$$\frac{\rho_2}{\rho_1} \cong 1 + \frac{z_1 - z_2}{H} . \tag{11}$$

Since $r_o >> H$, the density change in (11) is much greater than that given by Eq. (7). In other words, a rectilinear decay curve requires that the satellite face smaller densities than it would normally face in descending through a stationary atmosphere. This is possible only if densities decreased naturally as the satellite descended, i.e., if the atmosphere was undergoing compression during the descent.

If a decay curve possesses positive curvature over a considerable length of time, i.e., it exhibits concavity upwards (second derivative $\ddot{a} > 0$), then logically, the satellite is progressively facing even smaller densities than in a rectilinear decay. This, then signifies that the atmosphere was contracting at an even faster rate. To sum up, a decay curve that is straight or concave upwards indicates atmospheric compression. Decay curves which are concave downwards ($\ddot{a} > 0$), on the other hand, include all three possibilities of expanding, stationary or contracting atmospheres.

We now look into the significance of the second derivative $\ddot{a}$. Taking logarithm of Eq. (3) and differentiating with respect to time, we have

$$\frac{\ddot{a}}{\dot{a}} = \frac{\dot{a}}{a} + \frac{\dot{\rho}}{\rho} + \frac{\dot{v}}{v} . \tag{12}$$

Likewise, from Eq. (2)

$$\frac{\dot{v}}{v} = - \frac{\dot{a}}{2\,a} . \tag{13}$$

Combining (12) and (13) gives

$$\frac{\dot{\rho}}{\rho} = \frac{\ddot{a}}{\dot{a}} - \frac{\dot{a}}{2\,a} . \tag{14}$$

Geometrically speaking, $\ddot{a}$ determines the curvature of the decay curve: a positive curvature ($\ddot{a} > 0$) denotes concavity upwards, whereas a negative curvature ($\ddot{a} < 0$) denotes concavity downwards. The radius of curvature is given by the expression

$$R = \frac{(1 + \dot{a}^2)^{3/2}}{\ddot{a}} . \tag{15}$$

Substituting in Eq. (14), we get

$$\frac{\dot{\rho}}{\rho} = \frac{(1 + \dot{a}^2)^{3/2}}{\dot{a}\,R} - \frac{\dot{a}}{2\,a} . \tag{16}$$

For small but finite intervals, the left hand side of Eq. (16) can be taken as $\Delta\rho/\rho/\Delta t$, which represents the relative change in air density $\Delta\rho/\rho$ encountered by the satellite during a time interval Δt as it descends in the atmosphere. The second term on the right hand side is actually positive since $\dot{a} < 0$. The first term on the right is positive for negative curvature ($R < 0$, downward concavity) and negative for positive curvature ($R > 0$, upward concavity). Depending on the magnitude of R, the second term can dominate the first to determine the outcome of the left hand side. If the left hand side is negative, this means that the satellite is actually facing smaller densities as it descends. That occurs when

$$R < \frac{2\,a}{\dot{a}^2}\,(1 + \dot{a}^2)^{3/2} . \tag{17}$$

This is possible when the atmosphere is contracting at a faster rate than the rate of descent of the satellite. If the left hand side of Eq. (16) is positive, that merely indicates that the satellite is facing greater densities during descent. The atmosphere may even be contracting at the same time, but the satellite must be descending at a rate faster than the atmospheric contraction in the last case.

RESULTS AND DISCUSSIONS

This study analyzes the orbital decay of ODEARCS Spheres 1 and 6. Sphere 1 was a 4"-diameter chrome-plated Aluminum sphere having a mass of 1.488 kg, whereas Sphere 6 was a 6"-diameter sand-blasted Aluminum sphere of mass 5.000 kg. It was shown that chrome-plating Aluminum increased the drag coefficient by 8%, whereas sand-blasting increased it by 9% (Tan and Badhwar, 1995). However, since this difference is not significant, the traditional value of 2.2 for C_D was assumed for both spheres (cf. Cook, 1965).

Figure 1 (from The ODERACS News Roundup, December 1994; The ODERACS News Roundup, February, 1995) shows the relative orbital decay (mean altitude vs. time) of the ODERACS Spheres 1 and 6. The decay curve of Sphere 1

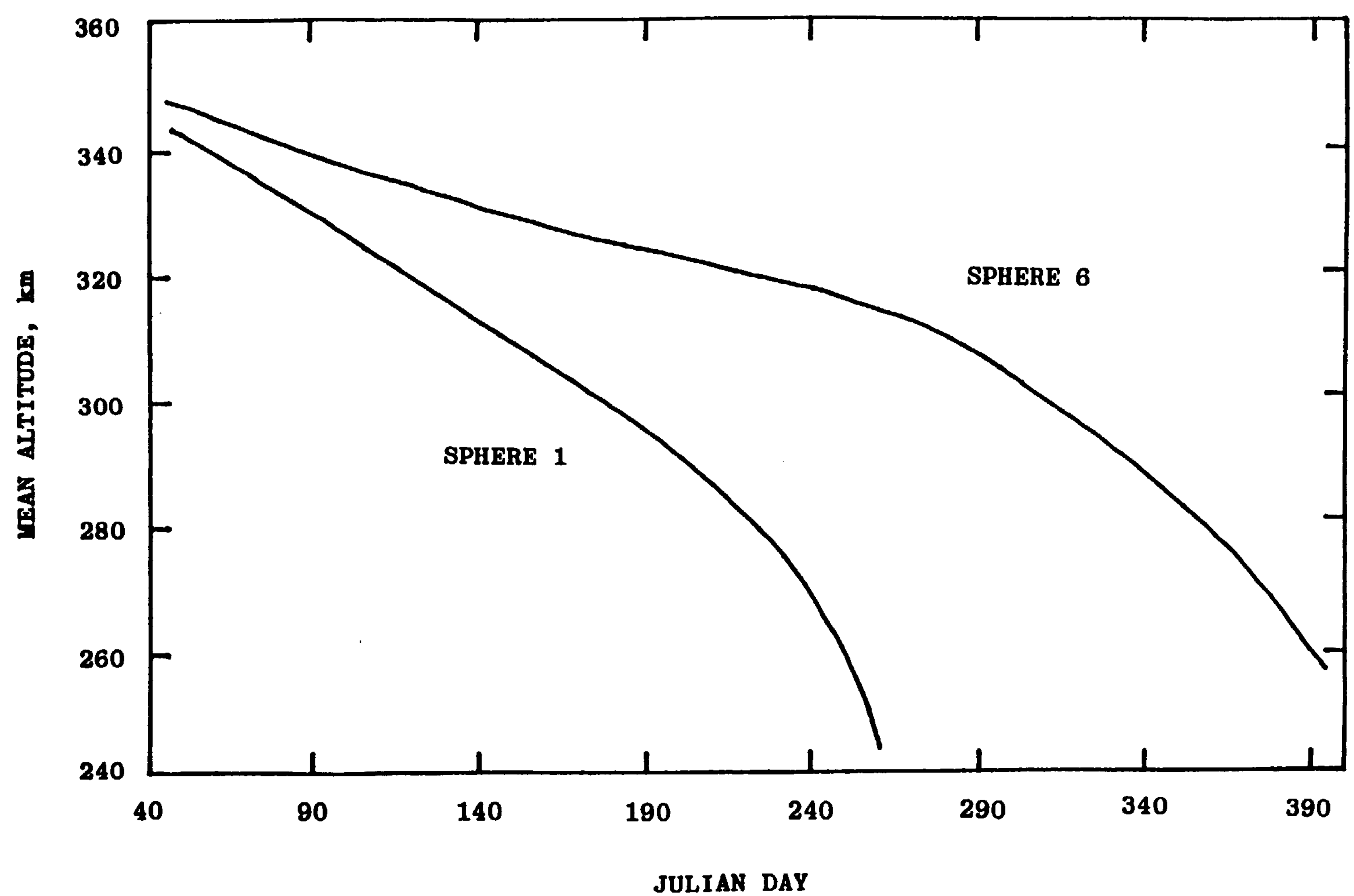

Fig. 1. Orbital Decay Curves (Mean Altitude vs. Day of Year) of ODERACS Spheres 1 and 6.

through Day 190, 1994 was almost straight with a slight concavity downwards. However, the most striking feauture lies on the decay curve of Sphere 6. Through Day 240, the curve exhibits a definite concavity upwards. According to our earlier analysis, this is the interval during which a compression of the atmosphere must have taken place.

The atmospheric densities encountered by ODERACS Spheres 1 and 6 were calculated at intervals of 50 days using Eq. (3) and entered in Tables I and II respectively. Sphere 1 had a greater rate of decay and consequently a shorter lifetime in the atmosphere owing to its larger area-to-mass ratio (A/m). It is remarkable that despite the drop of altitude of over 50 km in 150 days, the densities encountered by Sphere 1 in its early days remained nearly the same. Normally, there would have been a 3-fold increase in the density if the atmospheric conditions had remained the same (cf. U.S. Standard Atmosphere, 1976). This suggests that the atmosphere was undergoing compression during the same period.

A more definite proof of this compression comes from Table II. Through Day 240, Sphere 6 had actually encountered progressively smaller densities as it descended, which means that condition (17) must have been met during this period. In accordance with our earlier analysis, this confirms the fact that the atmosphere had contracted at a rate faster than the rate of descent of Sphere 6. The density encountered by the sphere decreased at an average rate of about 1.9% per 10 days. This rate of decrease can also be calculated from Eq. (16) and Fig. 2. The decay curve through Day 240 is nearly circular. The radius of curvature is estimated from a least-square fit circle and converted to proper units. The rate of relative decrease in density turns out to be 2.2% per 10 days, which fairly agrees with the earlier figure.

Next, since Spheres 1 and 6 descended over different altitudes at the same time, it is possible to calculate the average scale height of the atmosphere over the altitude range under consideration. Assuming an average scale height H, the densities ρ_1 at altitude z_1 (Sphere 1) and ρ_2 at altitude z_2 (Sphere 6) can be taken as those given by Eqs. (9) and (10) respectively. Here ρ_o is the density at a base level, where fluctuations due to solar 10.7 cm flux are a minimum. This level occurs at about 100 km (cf. U.S. Standard Atmosphere, 1976), which is well below the altitude range of interest. Eliminating ρ_o from Eqs. (9) and (10), we get

$$H = \frac{z_1 - z_2}{\ln \frac{\rho_2}{\rho_1}} . \tag{18}$$

The scale heights calculated from Eq. (18) using the data from Tables I and II are entered in Table III. It is interesting to note that there is a continuous decrease in the scale height during this period which cannot be accounted for by the much smaller normal decrease over the

Table I. Atmospheric Densities Encountered by Sphere 1

Julian Day	z, km	a, km	v, km/s	$\dot{a}$, 10^{-3} m/s	ρ, 10^{-12} kg/m^3
90	330.2	6708.3	7.708	3.90	6.29
140	313.3	6691.5	7.718	4.29	6.93
190	293.5	6671.7	7.730	5.36	8.67
240	267.2	6643.3	7.745	8.58	13.91

Table II. Atmospheric Densities Encountered by Sphere 6

Julian Day	z, km	a, km	v, km/s	$\dot{a}$, 10^{-3} m/s	ρ, 10^{-12} kg/m^3
90	339.8	6718.0	7.703	2.11	5.08
140	331.5	6709.6	7.708	1.84	4.43
190	324.3	6702.4	7.712	1.65	3.98
240	317.6	6695.7	7.716	1.51	3.65
290	307.4	6685.6	7.721	3.78	9.13
340	287.6	6665.7	7.733	5.15	12.44
390	259.4	6637.6	7.775	8.04	19.42

Table III. Atmospheric Scale Heights

Julian Day	z_1, km	z_2, km	$\rho_1, 10^{-12} kg/m^3$	$\rho_2, 10^{-12} kg/m^3$	H, km
90	330.2	339.8	6.291	5.079	45.0
140	313.3	331.5	6.929	4.429	40.6
190	293.5	324.3	8.674	3.977	39.4
240	267.2	317.6	13.910	3.651	37.7

same altitude range. This provides further evidence that from Day 90 through Day 240 during the year 1994, the atmosphere was in a compression mode.

Finally, it would be instructive to calculate the rate of contraction of the atmosphere during Days 90 through 240, 1994. One must bear in mind that the contraction rate would depend on the density level, but this dependence would be slight over the altitude range under consideration. We can calculate the altitude z corresponding to a certain density ρ from Eq. (8). Eliminating ρ_0 between (8) and (9) or (10), we get

$$z = -H \ln \left(\frac{\rho}{\rho_1} e^{-\frac{z_1}{H}}\right) = -H \ln \left(\frac{\rho}{\rho_2} e^{-\frac{z_2}{H}}\right) \quad . \tag{19}$$

The levels of constant density equal to 5×10^{-12} kg/m^3 are calculated using Eq. (19) and plotted in Fig. 3 along with the altitudes of descent of Spheres 1 and 6. The figure shows that this level descended steadily at a rate of approximately 2.3 km per 10 days. The figure further verifies our earlier findings that Sphere 1 and Sphere 6 descended faster and slower, respectively, than this level of constant density. Consequently, Sphere 6 encountered smaller densities while Sphere 1 encountered greater densities even as the atmosphere itself was undergoing compression at the same time.

This entire analysis could not be extended beyond Day 240, for Sphere 1 had deorbited on Day 275 (The ODERACS News Roundup, December 1994) and we cannot calculate the scale height from the orbital data of Sphere 6 alone. However, we can still determine the dynamical state of the atmosphere subsequent to Day 240 as follows. If ρ_1 is the density at altitude z_1, we can calculate the density ρ_2 at a lower altitude z_2 if the satellite descended in a stationary atmosphere by assuming a constant plausible scale height H. From Eqs. (9) and (10)

$$\rho_2 = \rho_1 e^{\frac{z_1 - z_2}{H}} \quad . \tag{20}$$

If Sphere 6 had descended from 317.6 km to 307.4 km between Day 240 to Day 290 (cf. Table II) in a stationary atmosphere, then the density at the latter height would range from 4.58×10^{-12} kg/m^3 to 5.13×10^{-12} kg/m^3 corresponding to scale heights between 45 km and 30 km. Since this range of density is only about half of the observed density (9.13×10^{-12} kg/m^3), the satellite had actually encountered far greater densities than it would have in a stationary atmosphere. This necessarily means that the atmosphere was in an expansion mode from Day 240 to Day 290. Similar analyses carried out between Day 290 and Day 340 and between Day 340 and Day 390 indicate that the atmosphere had reverted to a compression mode during both of these periods. In summation, the atmosphere was in a compression mode from Day 90 through Day 140

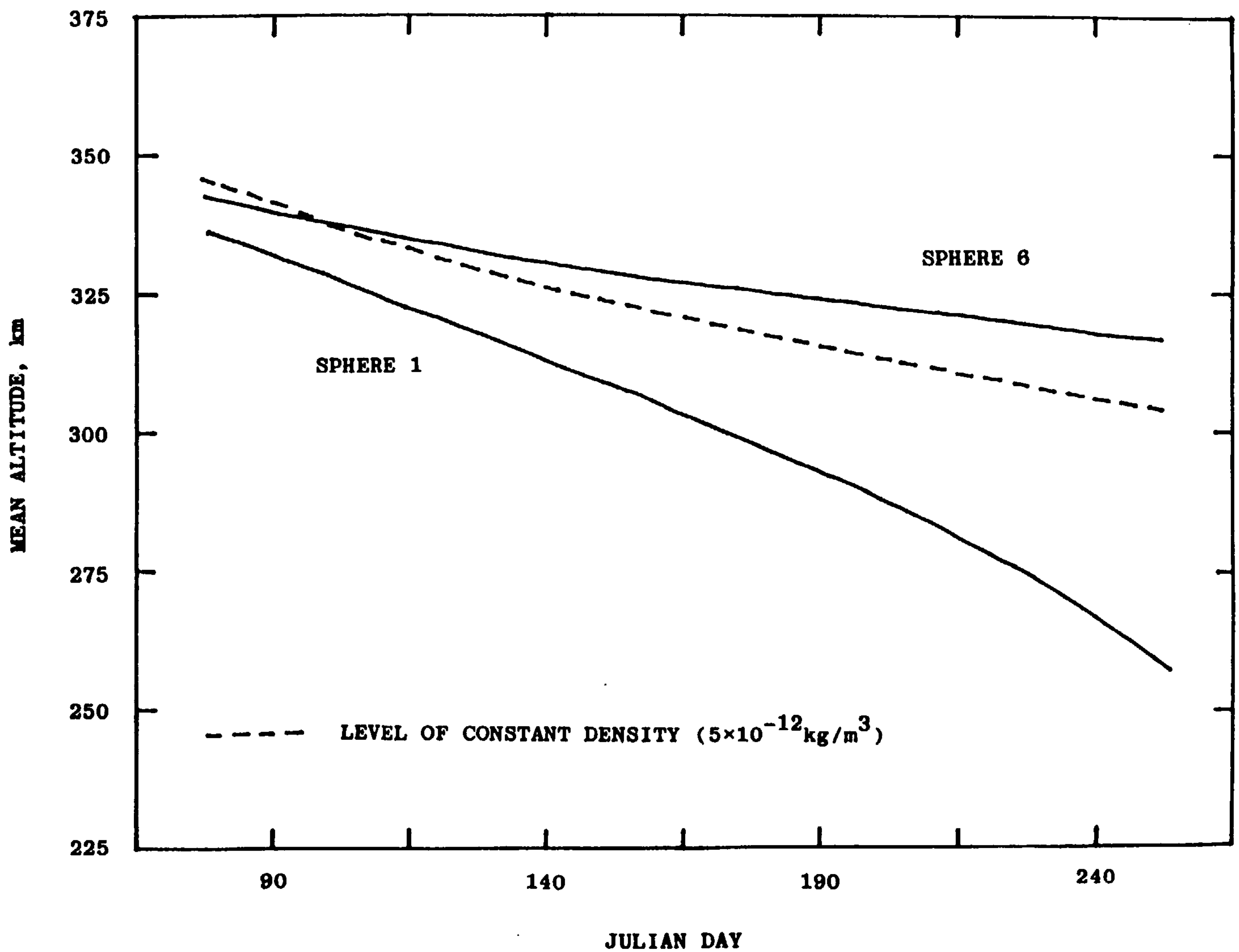

Fig. 2. Altitudinal descent of ODERACS Spheres 1 and 6 and atmospheric level of constant density

followed by an expansion mode from Day 240 through Day 290 and finally, another compression mode from Day 290 through Day 390 during the year 1994.

CONCLUSIONS

The following conclusions are drawn from the analysis of the orbital decay of satellites in circular orbits.
(1) A rectilinear decay curve (zero curvature) indicates atmospheric compression.
(2) A decay curve with concavity upwards (positive curvature) strongly indicates atmospheric contraction.
(3) A decay curve with downward concavity (negative curvature) may indicate expanding, stationary or contracting atmospheres.
(4) Throughout the period from Day 90 through Day 240 during the year 1994, the atmosphere was in a compression mode.
(5) During this period, ODERACS Sphere 1 faced nearly constant densities while Sphere 6 actually encountered progressively smaller densities as they descended.
(6) The atmospheric scale height diminished steadily during the same period.
(7) During a brief period from Day 240 to Day 290, the atmosphere reversed to a strongly expanding mode.
(8) The atmosphere reverted back to a compression mode from Day 290 through Day 390, 1994.

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